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INTERACTION OF A PLANE HARMONIC RAYLEIGH WAVE WITH A THIN RIGID EDGE INCLUSION COUPLED WITH AN ELASTIC MEDIUM†

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Oscillations of a rigid edge inclusion placed in an elastic half-plane, coupled with an elastic medium and extending onto a surface perpendicular to it, are considered. The oscillations are induced by a plane harmonic Rayleigh surface wave propagating in the elastic medium. To solve this problem, the displacement field in the half-plane is expressed as a superposition of the displacements induced by the propagating Rayleigh wave and two discontinuous solutions of the Lamé equations with jumps at the boundary of the half-plane and at the line along which the inclusion is situated. The unknown jumps are determined from the boundary conditions and the conditions of the interaction of the inclusion with the medium. This reduces the problem to the solution of a system of singular integral equations, with a stationary singularity, for the jumps of the stresses on the line of the inclusion. The system is solved numerically by mechanical quadratures. The parameters of the motion of the inclusion and the stressed state of the medium near it are investigated. © 1997 Elsevier Science Ltd. All rights reserved.

1. Consider an elastic half-plane $-\infty < x < \infty, y > 0$, whose boundary is stress-free

$$\sigma_y(x, +0) = \tau_{yx}(x, +0) = 0, \quad -\infty < x < \infty \tag{1.1}$$

and which contains a thin rigid inclusion situated in the closed interval $x = 0, 0 \leqslant y \leqslant h$. The stress is discontinuous across the line on which the inclusion is situated, with jumps

$$\begin{aligned} \langle\sigma_x\rangle &= \sigma_x(+0, y) - \sigma_x(-0, y) = \chi_1(y) \\ \langle\tau_{xy}\rangle &= \tau_{xy}(+0, y) - \tau_{xy}(-0, y) = \chi_2(y) \end{aligned} \tag{1.2}$$

A plane harmonic Rayleigh wave [1], propagating along the positive direction of the x axis, produces displacements in the medium

$$\begin{aligned} u_R(x,y) &= C\exp(i\varkappa_R x)\left[\exp(-\varkappa_2 g_1 y) - \frac{\mu_R g_2}{\xi_R}\exp(-\varkappa_2 g_2 y)\right] \\ v_R(x,y) &= iC\exp(i\varkappa_R x)\left[\xi_R^{-1} g_1\exp(-\varkappa_2 g_1 y) - \mu_R\exp(-\varkappa_2 g_2 y)\right] \end{aligned} \tag{1.3}$$

$$g_1 = \sqrt{\xi_R^2 - \xi^2}, \quad g_2 = \sqrt{\xi_R^2 - 1}, \quad g_3 = 2\xi_R^2 - 1, \quad \xi = c_2/c_1, \quad \xi_R = c_2/c_R$$

$$\mu_R = 2\xi_R g_1/g_3, \quad \varkappa_R = \omega/c_R, \quad \varkappa_j = \omega/c_j, \quad j = 1,2$$

where c_1, c_2 and c_R are the velocities of longitudinal, transverse and surface waves, respectively, in the medium, and ω is the oscillation frequency. In view of the linearity, we will henceforth omit the time factor $\exp(-i\omega t)$.

We will assume that the inclusion is coupled with the elastic medium. Then the following conditions must hold on the position line of the inclusion

$$u(\pm 0, y) = \delta_1 + \gamma y, \quad v(\pm 0, y) = \delta_2, \quad y \in [0, h] \tag{1.4}$$

where δ_1 and δ_2 are the horizontal and vertical displacements and γ is the angle through which the inclusion rotates. The quantities δ_1, δ_2, γ may be determined from the equations of motion of the inclusion

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$$m\ddot{x}_c = F_x e^{-i\omega t}, \quad m\ddot{y}_c = F_y e^{-i\omega t}, \quad J_c \ddot{\varepsilon} = M e^{-i\omega t} \tag{1.5}$$

where x_c and y_c are the displacements of the centre of gravity of the inclusion, ε is the angular acceleration, m is the mass of the inclusion, J_c is its moment of inertia about an axis through the centre of gravity, and F_x, F_y and M are the forces and moment exerted on the inclusion by the medium, which are defined by the formulae

$$F_x = \int_0^h \chi_1(y)dy, \quad F_y = \int_0^h \chi_2(y)dy, \quad M = \int_0^h y\chi_1(y)dy \tag{1.6}$$

The displacements in the elastic half-plane are represented as

$$u = u_1 + u_2 + u_R, \quad v = v_1 + v_2 + v_R \tag{1.7}$$

where u_1 and v_1 are solutions of the Lamé equations with jumps (1.2), and u_2 and v_2 are solutions of the Lamé equations at the boundary of the half-plane

$$[\sigma_y] = \phi_1(x), \quad [\tau_{yx}] = \phi_2(x), \quad [v] = \phi_3(x), \quad [u] = \phi_4(x) \tag{1.8}$$

$$[f] = f(x, +0) - f(x, -0)$$

These solutions are given by formulae (2) in [2] and (1.2) in [3].

The jumps $\phi_k(x)$ ($k = 1, 2, 3, 4$) occurring in v_2 and u_2 may be determined from conditions (1.1). But as these conditions are not sufficient to determine all four jumps, we need two additional conditions. They may be obtained by requiring that

$$u_2(x, -0) = v_2(x, -0) = 0$$

It then follows from definition (1.8) of the jumps that

$$[v_2] = v_2(x, +0) = \phi_3(x), \quad [u_2] = u_2(x, +0) = \phi_4(x) \tag{1.9}$$

Substituting u_2 and v_2 from [2, 3], taking Fourier transforms with respect to x and applying the convolution theorem, we obtain

$$\begin{aligned}
&\Phi_3(\alpha) = R(\alpha)^{-1}\left[\varkappa_2^2\gamma_1(\alpha)\Phi_1(\alpha) + (-i\alpha)B(\alpha)\Phi_2(\alpha)\right] \\
&\Phi_4(\alpha) = R(\alpha)^{-1}\left[\varkappa_2^2\gamma_2(\alpha)\Phi_2(\alpha) - (-i\alpha)B(\alpha)\Phi_1(\alpha)\right] \\
&R(\alpha) = (2\alpha^2 - \varkappa_2^2)^2 - 4\alpha^2\gamma_1(\alpha)\gamma_2(\alpha), \quad B(\alpha) = 2\alpha^2 - \varkappa_2^2 - 2\gamma_1(\alpha)\gamma_2(\alpha) \\
&\gamma_j(\alpha) = \sqrt{\alpha^2 - \varkappa_j^2}, \quad j = 1, 2
\end{aligned} \tag{1.10}$$

where $\Phi_k(\alpha)$ are the Fourier transforms of the jumps.

Formulae (1.10) yield

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \iint_{-\infty}^{\infty} \frac{\exp(i\alpha(\eta - x))}{R(\alpha)} \begin{pmatrix} \gamma_1(\alpha)d_3(y,\alpha) & (-i\alpha)d_1(y,\alpha) \\ (-i\alpha)d_2(y,\alpha) & \gamma_2(\alpha)d_4(y,\alpha) \end{pmatrix} \begin{bmatrix} \phi_1(\eta) \\ \phi_2(\eta) \end{bmatrix} d\alpha d\eta$$

$$d_1(y,\alpha) = (2\alpha^2 - \varkappa_2^2)e^{-\gamma_1 y} - 2\gamma_1\gamma_2 e^{-\gamma_2 y}, \quad d_2(y,\alpha) = (2\alpha^2 - \varkappa_2^2)e^{-\gamma_2 y} - 2\gamma_1\gamma_2 e^{-\gamma_1 y}, \tag{1.11}$$

$$d_3(y,\alpha) = (2\alpha^2 - \varkappa_2^2)e^{-\gamma_1 y} - 2\alpha^2 e^{-\gamma_2 y}, \quad d_4(y,\alpha) = (2\alpha^2 - \varkappa_2^2)e^{-\gamma_2 y} - 2\alpha^2 e^{-\gamma_1 y}$$

We now substitute (1.7) into boundary conditions (1.1). Taking (1.11) into consideration, as well as the expressions for u_1 and v_1 in [2], we obtain the following equalities

$$\phi_1(x) = -\frac{1}{2\pi\varkappa_2^2}\int_0^h \chi_1(\eta)\int_{-\infty}^{\infty} \frac{(-i\alpha)e^{-i\alpha x}}{2\gamma_1(\alpha)} d_1(\alpha,\eta)d\alpha d\eta - \frac{1}{4\pi\varkappa_2^2}\int_0^h \chi_2(\eta)\int_{-\infty}^{\infty} d_3(\eta,\alpha)e^{-i\alpha x}d\alpha d\eta \tag{1.12}$$

$$\phi_2(x) = -\frac{1}{4\pi\varkappa_2^2}\int_0^h \chi_1(\eta)\int_{-\infty}^{\infty} d_4(\alpha,\eta)e^{-i\alpha x}d\alpha d\eta - \frac{1}{4\pi\varkappa_2^2}\int_0^h \chi_2(\eta)\int_{-\infty}^{\infty} \frac{(-i\alpha)e^{-i\alpha x}}{\gamma_2(\alpha)} d_2(\alpha,\eta)d\alpha d\eta$$

In deriving these formulae we have used the fact that the following conditions hold for (1.11)

$$\sigma_y^2(x,+0) = \phi_1(x), \quad \tau_{yx}^2(x,+0) = \phi_2(x)$$

The final representation of u_2 and v_2 in terms of the jumps $\chi_1(y)$ and $\chi_2(y)$ now follows from (1.11) and (1.12)

$$v_2(x,y) = \sum_{j=1}^{2}\int_0^h \chi_j(\eta)D_{1j}(\eta,y,x)d\eta$$
$$u_2(x,y) = \sum_{j=1}^{2}\int_0^h \chi_j(\eta)D_{2j}(\eta,y,x)d\eta \tag{1.13}$$

where

$$D_{kj}(\eta,y,x) = \frac{1}{2\pi\varkappa_2^2}\int_{-\infty}^{\infty}\frac{e^{-i\alpha x}}{R(\alpha)}F_{kj}(\alpha,\eta,y)d\alpha, \quad k=1,2; \quad j=1,2$$

and the functions $F_{kj}(\alpha, \eta, y)$ are expressed in terms of $d_m(\alpha, y)$ and $d_m(\alpha, \eta)$ ($m = 1, 2, 3, 4$).

2. Formulae (1.7) and (1.3) enable us to express the displacement and stress field and in the elastic half-plane in terms of the unknown jumps of the stresses at the inclusion. To determine them, we will use conditions (1.4), after first differentiating them in order to eliminate the unknown constants δ_1 and δ_2. Conditions (1.4) then become

$$\frac{\partial u}{\partial y}(\pm 0,y) = \gamma, \quad \frac{\partial v}{\partial y}(\pm 0,y) = 0, \quad y\in[0,h] \tag{2.1}$$

These equalities will be equivalent to (1.4) if we add the further conditions

$$u(\pm 0, 0) = \delta_1, \quad v(\pm 0, 0) = \delta_2 \tag{2.2}$$

Substituting (1.7) into (2.1) and (2.2), we obtain a system of integral equations for the unknown jumps in the stresses at the inclusion

$$\int_0^1 \omega_j(\tau)[s_j(\tau-t)+K_j(\tau,t)]d\tau = p_j(t), \quad t\in[0,1]$$
$$\int_0^1 \omega_j(\tau)[s_{0j}(\tau)+K_j(\tau,0)]d\tau = \delta_{0j}-p_{0j}, \quad j=1,2 \tag{2.3}$$

where we have used the following notation

$$\tau = h^{-1}\eta, \quad t = h^{-1}y, \quad \omega_j(\tau) = \mu^{-1}\chi_j(h\tau), \quad \delta_{0j} = h^{-1}\delta_j, \quad j=1,2, \quad \varkappa_0 = \varkappa_2 h$$

The kernels of the integral operators in (2.3) are as follows:

$$s_{0j}(\tau-t) = \frac{1}{4\pi}\int_{-\infty}^{\infty} L_j(\beta)\exp(i\beta\varkappa_0(\tau-t))d\beta, \quad L_j(\beta) = \lambda_j^{-1}(\lambda_1\lambda_2-\beta^2)$$

$$s_j(\tau-t) = \frac{\partial}{\partial t}s_{0j}(\tau-t), \quad j=1,2; \quad \lambda_1 = \sqrt{\beta^2-\xi^2}, \quad \lambda_2 = \sqrt{\beta^2-1}$$

$$K_{0j}(\tau,t) = \frac{1}{4\pi}\int_{-\infty}^{\infty} W_j(\beta,\tau,t)d\beta, \quad K_j(\tau,t) = \frac{\partial}{\partial t}K_{0j}(\tau,t) \tag{2.4}$$

$$W_j(\beta,\tau,t) = R_*(\beta)^{-1}A_j(\beta,\tau,t)$$

$$A_1(\beta,\tau,t)=\varkappa_2^{-5}F_{12}(\varkappa_2\beta,h\tau,ht),\quad A_2(\beta,\tau,t)=\varkappa_2^{-5}F_{21}(\varkappa_2\beta,h\tau,ht)$$

$$R_*(\beta)=\varkappa_2^{-4}R(\varkappa_2\beta)=(2\beta^2-1)^2-4\beta^2\lambda_1\lambda_2$$

The right-hand sides of (2.3) are defined as follows:

$$p_1(t)=\gamma+\frac{\partial u_R}{\partial y}(0,ht),\quad p_2(t)=\frac{\partial v_R}{\partial y}(0,ht),\quad p_{01}=h^{-1}u_R(0,0),\quad p_{02}=h^{-1}v_R(0,0)$$

Let us investigate the properties of the functions defined in (2.4). We first consider the functions s_{0j} and s_j. In view of the asymptotic expansion

$$L_j(\beta)=-\frac{(1+\xi^2)}{2}|\beta|^{-1}+O(\beta^{-3}),\quad \beta\to\infty$$

it follows from the integral representation in (2.4) that

$$\begin{aligned}&s_{0j}(z)=-\frac{(1+\xi^2)}{2}\ln|z|+O(z^2\ln|z|)\\&s_j(z)=-\frac{1+\xi^2}{2z}+s_{1j}(z),\quad s_{1j}(z)=O(z\ln|z|),\quad z\to 0\end{aligned}\tag{2.5}$$

Now consider the functions $K_j(\tau, t)$. In the integral of (2.4) representing these functions, we introduce attenuation in the medium and change to contour integration in the complex plane of $\beta = \sigma + i\zeta$ (see [1]). We then pass to the limit of a medium without attenuation and obtain

$$K_{0j}(\tau,t)=\frac{1}{4\pi}\left\{2\pi i\frac{A_j(\xi_R,\tau,t)}{R_*'(\xi_R)}-\sum_{k=1}^{2}\int_{L_k}\frac{A_j(\beta,\tau,t)}{R_*(\beta)}d\beta\right\},\quad j=1,2$$

Cut the plane along a curve L_1 from the point $\beta = \xi$, consisting of the segment of the real axis $0 < \sigma < \xi$ and the positive imaginary axis $\zeta > 0$. Make a further cut L_2 from the point $\beta = 1$, consisting of the segment of the real axis $0 < \sigma < 1$ and the positive imaginary axis $\zeta > 0$. These cuts are defined by the choice of a single-valued branch of the functions $\lambda_j(\beta)$ in accordance with (1.12).

Transforming the integrals along the cuts, we obtain

$$\begin{aligned}&K_j(\tau,t)=i\frac{A_j'(\xi_R,\tau,t)}{R_*'(\xi_R)}+K_{1j}(\tau,t)+iK_{2j}(\tau,t)+iK_{3j}(\tau,t)\\&K_{1j}(\tau,t)=\frac{\varkappa_0}{2\pi}\int_0^\infty\frac{V_1(\zeta,\tau,t)}{R_4(\zeta)}d\zeta,\quad K_{2j}(\tau,t)=-\frac{\varkappa_0}{2\pi}\int_0^\xi\frac{V_2(\sigma,\tau,t)}{R_3(\sigma)}d\sigma\\&K_{3j}(\tau,t)=-\frac{\varkappa_0}{2\pi}\int_\xi^1\frac{V_{3j}(\sigma,\tau,t)}{R_0(\sigma)}d\sigma,\quad A_j'(\beta,\tau,t)=\frac{\partial}{\partial t}A_j(\beta,\tau,t)\\&R_0(\beta)=(\lambda_3^-)^4+16\beta^4(\lambda_1^+\lambda_2^-)^2,\quad R_3(\beta)=(\lambda_3^-)^2-4\beta^2\lambda_1^-\lambda_2^-\\&R_4(\beta)=(\lambda_3^+)^2-4\beta^2\lambda_1^+\lambda_2^+,\quad \lambda_3^\pm=2\beta^2\pm1\\&\lambda_1^\pm=\sqrt{\xi^2\pm\beta^2},\quad \lambda_2^\pm=\sqrt{1\pm\beta^2}\end{aligned}\tag{2.6}$$

The functions $V_{kj}(\beta, \tau, t)$, ($k = 1, 2, 3; j = 1, 2$) are expressed in terms of their values at the edges of the cuts, $A'_j(\beta, \tau, t)$.

It can be shown that the first and last terms in (2.6) are bounded as $\tau, t \to 0$, and that $K_{2j}(\tau, t) = O(\tau + t))$.

Let us investigate the behaviour as $\tau, t \to 0$ of the function $K_{1j}(\tau, t)$. The integrand in (2.6) that defines $K_{1j}(\tau, t)$ admits of the following asymptotic expansion as $\tau, t \to 0$, $\zeta \to \infty$

$$\frac{V_{1j}(\zeta,\tau,t)}{R_4(\zeta)} = V_{0j}(\zeta,\tau,t) + O(\zeta^{-2})$$

$$V_{0j}(\zeta,\tau,t) = -2\varkappa_0 \sum_{k=0}^{3} B_k(\tau,t)\zeta^{2-k}\sin(\zeta\varkappa_0(\tau+t)+\frac{1}{2}k\pi) \tag{2.7}$$

$$B_{01}(\tau,t) = B_{02}(\tau,t) = \varkappa_0^2(1-\xi^2)\tau t$$

$$B_{11}(\tau,t) = -\tfrac{1}{2}\varkappa_0[(3-\xi^2)\tau + (1-\xi^2)t] + O((\tau+t)^2)$$

$$B_{12}(\tau,t) = -\tfrac{1}{2}\varkappa_0[2(1-\xi^2)\tau + (\xi^4-1)(\tau+t)] + O((\tau+t)^2)$$

$$B_{21} = (1-\xi^2) + O((\tau+t)), \quad B_{22} = \xi^4 + O((\tau+t))$$

$$B_{31} = B_{32} = O((\tau + t))$$

It follows from (2.7) that the integrals defining $K_{1j}(\tau, t)$ are divergent. To give them a definite meaning, we transform them as follows:

$$K_{1j}(\tau,t) = K_{1j}^0(\tau,t) + K_{1j}^1(\tau,t)$$

$$K_{1j}^0(\tau,t) = \frac{\varkappa_0}{2\pi}\int_0^\infty V_{0j}(\zeta,\tau,t)d\zeta \tag{2.8}$$

$$K_{1j}^1(\tau,t) = \frac{\varkappa_0}{2\pi}\int_0^\infty \left[\frac{V_{1j}(\zeta,\tau,t)}{R_4(\zeta)} - V_{0j}(\zeta,\tau,t)\right]d\zeta$$

The functions $K_{1j}^0(\tau, t)$ are given by divergent integrals whose values may be defined using the theory of generalized functions [4]. Doing so, we deduce from (2.7) and (2.8) that

$$2\pi K_{1j}^0(\tau,t) = \frac{(1-\xi^2)\tau(\tau-t)}{(\tau+t)^3} + \frac{(1+\xi^4)}{2(1-\xi^2)(\tau+t)} + O((\tau+t)), \quad j = 1,2 \tag{2.9}$$

In view of (2.5) and (2.9), the integral equation (2.3) may be written in the form

$$\int_0^1 \omega_j(\tau)\left[\frac{1}{\tau-1} + \frac{a}{\tau+t} + b\frac{\tau(\tau-t)}{(\tau+t)^3} + h_j(\tau-t) + B_j(\tau,t)\right]d\tau = f_j(t) \tag{2.10}$$

where

$$f_j(t) = -\frac{\pi}{1+\xi^2}p_j(t), \quad h_j(z) = O(z\ln|z|), \quad z \to 0$$

$$B_j(\tau,t) = O(1), \quad \tau,t \to 0, \quad a = -\frac{1+(3-4\nu)^2}{2(3-4\nu)}, \quad b = \frac{2}{3-4\nu}$$

(ν is Poisson's ratio).

Equation (2.10) is a singular integral equation with a stationary singularity. It has been proved [5] that this equation is Noetherian in the class $L_1(I)$, $I = [0, 1]$ and of index one. If $f_j \equiv 0$, the equation has only one linearly independent solution

$$\omega_j(\tau) = \tau^\alpha + a_j(1-\tau)^{-\frac{1}{2}} + \omega_{0j}(\tau), \quad j = 1,2 \tag{2.11}$$

where $\omega_{0j}(\tau)$ is bounded for $\tau \in [0, 1]$, and α ($-1 < \alpha < 0$) is a root of the transcendental equation

$$\cos \pi\alpha - a + b(1+\alpha)^2 = 0$$

In particular, if $\nu = 0.25$ then $\alpha = -0.22525$.
In accordance with (2.11), we look for an approximate solution of (2.3) as

$$\omega_j(\tau) = \tau^{\alpha}(1-\tau)^{-1/2}\psi_j(\tau) \tag{2.12}$$

To determine an approximation for $\psi_j(\tau)$, after replacing the integrals in (2.3) by suitable quadrature formulae [6, 7], we obtain a system of linear algebraic equations

$$\sum_{k=1}^{n} w_k \psi_{jk}[s_j(\tau_k - t_m) + K_j(\tau_k, t_m)] = p_{jm}, \quad m = 1,2,\ldots,n-1 \tag{2.13}$$

$$\sum_{k=1}^{n} w_k \psi_{jk}[s_{0j}(\tau_k) + K_{0j}(\tau_k, 0)] = \delta_{0j} - p_{0j}, \quad j = 1,2$$

where we have used the following notation: $\psi_{jk} = \psi_j(\tau_k)$, $p_{jm} = p_j(t_m)$, $\tau_k = (1-x_k)/2$, x_k are the roots of the Jacobi polynomials $P_n^{\alpha,-1/2}(x)$, $t_m = (1-z_m)/2$, z_m are the roots of the Jacobi functions of the second kind

$$G_n(z) = \int_{-1}^{1} \frac{(1-x)^{\alpha}(1+x)^{-1/2}}{x-z} P_n^{\alpha,-1/2}(x)dx$$

$w_k = 2^{\alpha+1/2}A_k$, and A_k are the coefficients of the Gauss–Jacobi quadrature formula [7] with weight function $(1-x)^{\alpha}(1+x)^{-1/2}$.

To determine the unknown constants γ, δ_{0j} ($j = 1, 2$) in (2.3) which describe the motion of the inclusion interacting with the wave, we replace the integrals in (1.6) by suitable quadrature formulae and then deduce the following three equalities from (1.5) and (1.6)

$$\delta_{01} + \frac{\gamma}{2} = -\frac{1}{\varkappa_0^2 m_0}\sum_{k=1}^{n} w_k \psi_{1k}, \quad \gamma = -\frac{12}{\varkappa_0^2 m_0}\sum_{k=1}^{n} w_k \tau_k \psi_{1k} \tag{2.14}$$

$$\delta_{02} = -\frac{1}{\varkappa_0^2 m_0}\sum_{k=1}^{n} w_k \psi_{2k}, \quad m_0 = \frac{m}{\rho h^2}$$

After solving system (2.13) together with (2.14), we can approximate the functions $\psi_j(\tau)$ by interpolation polynomials

$$\psi_j(\tau) = Q_{nj}(\tau) = -\frac{1}{2}\sum_{k=1}^{n} \psi_{jk} \frac{P_n^{\alpha,-1/2}(1-2\tau)}{(\tau-\tau_k)\left[P_n^{\alpha,-1/2}(1-2\tau_k)\right]'} \tag{2.15}$$

Using (2.15), we can determine the contact stresses in the contact zone of the inclusion and the medium

$$\mu^{-1}\sigma_x(\pm 0, ht) = \pm\frac{\omega_1(t)}{2} = \pm q_1(t), \quad \mu^{-1}\tau_{xy}(\pm 0, ht) = \pm\frac{\omega_2(t)}{2} = \pm q_2(t)$$

By (2.12) and (2.15), we obtain

$$q_j(t) = \tfrac{1}{2} t^{\alpha}(1-t)^{-1/2} Q_{nj}(t), \quad j = 1,2 \tag{2.16}$$

The stressed state of the medium near the immersed end of the inclusion is characterized by the following stress intensity factors [8, 9]

$$K_1 = \lim_{y\to h+0}\left(\frac{y}{h}-1\right)^{1/2}\sigma_y(0,y), \quad K_2 = \lim_{y\to h+0}\left(\frac{y}{h}-1\right)^{1/2}\tau_{xy}(0,y)$$

Evaluating the limits, we find that

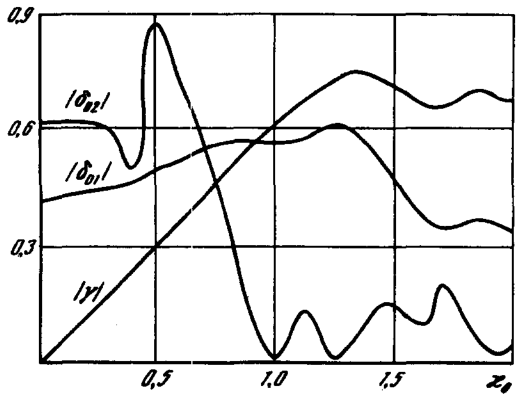

Fig. 1.

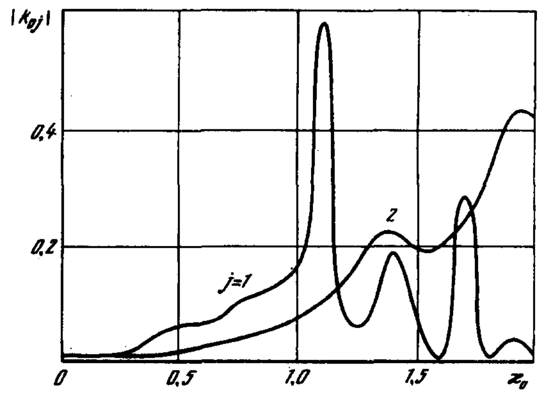

Fig. 2.

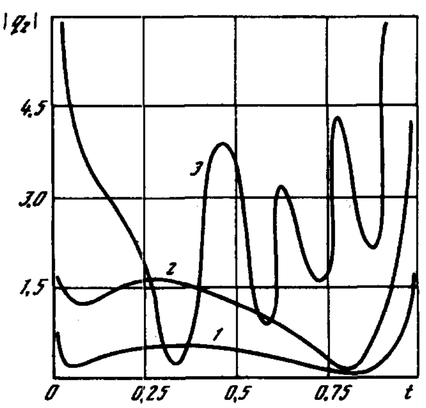

Fig. 3.

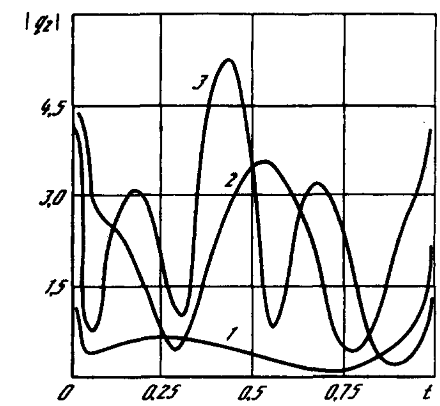

Fig. 4.

$$K_j = \mu k_{0j}, \quad j = 1,2, \quad k_{01} = \frac{1}{2} Q_{2n}(1), \quad k_{02} = \frac{\xi^2}{2} Q_{1n}(1) \tag{2.17}$$

where $Q_{jn}(1)$ are calculated from (2.15).

The approximate solution obtained here as used in a computation for the following data: $\nu = 0.25$, $m_0 = 1$, $c_0 = c/h = 1$. To obtain system (2.13), the quadrature formulae were used with up to 25 grid points, which was sufficient to obtain results with a relative error of less than 1%. The results of the numerical test are shown in Figs 1–4.

Figure 1 shows graphs against the parameter x_0 of the quantities $|\delta_{01}|$, $|\delta_{02}|$, $|\gamma|$, which describe the motion of the inclusion in the elastic medium. It can be seen that as x_0 increases the amplitude of the horizontal oscillations, $|\delta_{01}|$, first increases, reaches a maximum value at $x_0 = 1.3$ and then begins to decrease, through a sequence of minimum and maximum points. The quantity $|\gamma|$ behaves similarly. Not so the amplitude $|\delta_{02}|$ of the vertical oscillations. At first it decreases slowly, but then it has a sharp maximum at $x_0 = 0.5$, subsequently behaving in a rather complicated manner, with a succession of maxima and minima.

Figure 2 shows the absolute values of the stress intensity factors $|k_{01}|$, $|k_{02}|$ defined by (2.17) plotted against x_0. These plots also show several maxima at certain frequency values.

Figures 3 and 4 show the distribution of the absolute values of the contact stresses, $|q_1(t)|$ and $|q_2(t)|$, along the inclusion. Curves 1–3 correspond to $x_0 = 0.5$, 1 and 2. It is clear that the distribution of contact stresses depends essentially on the frequency of the propagating wave.

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